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THE UNSTEADY AXISYMMETRIC CONTACT PROBLEM WITH HEAT GENERATION†

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Unlike previous papers [1–3], where steady contact problems of thermoelasticity were investigated, an unsteady contact problem for a stiff finite cylinder with a plane base is considered. The cylinder is clamped to an elastic half-space and is rotated around its axis with constant angular velocity. Heat generation due to friction over the contact area, non-ideal thermal contact between the bodies and heat exchange with the surroundings from the free surfaces is assumed. A Laplace transformation with respect to the time coordinate, a Hankel transformation with respect to the radial coordinate for the half-space and the method of straight lines for the cylinder are used to solve the problem. The temperature and thermal flux fields in the cylinder and in the half-space, the contact stresses and the displacements of the half-space are determined. The problem is analysed for values of the input parameters which do not allow any change with time in boundary conditions.

1. FORMULATION OF THE PROBLEM

Consider a cylindrical punch of radius R and height l, which is indented with a vertical force P into an elastic half-space and is rotated with constant angular velocity ω (Fig. 1). At the initial instant of time we will assume the temperature of both bodies and the residue of the punch to be zero. The mathematical formulation of the problem is as follows:

the equations of thermoelasticity for the half-space

$$\Delta u_r - \frac{u_r}{r^2} + k\frac{\partial e}{\partial r} = \frac{\beta}{\mu}\frac{\partial t}{\partial r}, \quad \Delta u_z + k\frac{\partial e}{\partial z} = \frac{\beta}{\mu}\frac{\partial t}{\partial z} \tag{1.1}$$

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}, \quad e = \left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right)$$

the equations of heat conduction for the cylinder and the half-space

$$\Delta T = X_1\frac{\partial T}{\partial \tau}, \quad \Delta t = X_2\frac{\partial t}{\partial \tau} \tag{1.2}$$

the temperature boundary conditions

$$z = 0: \frac{\partial T}{\partial z} = \gamma_0 T \quad (0 \leqslant r \leqslant R); \quad r = R: \frac{\partial T}{\partial r} = -\gamma_\alpha T \quad (0 \leqslant z \leqslant l) \tag{1.3}$$

$$z = l: \lambda_2\frac{\partial t}{\partial z} - \lambda_1\frac{\partial T}{\partial z} = f_T\omega r\sigma_{zz}, \quad \lambda_2\frac{\partial t}{\partial z} + \lambda_1\frac{\partial T}{\partial z} = h(t - T) \quad (0 \leqslant r \leqslant R) \tag{1.4}$$

$$\frac{\partial t}{\partial z} = \gamma_H t \quad (r > R)$$

the force boundary conditions

$$z = l: u_z = f\ (0 \leqslant r \leqslant R), \quad \sigma_{zz} = 0\ (r \geqslant R), \quad \tau_{rz} = 0\ (r < \infty) \tag{1.5}$$

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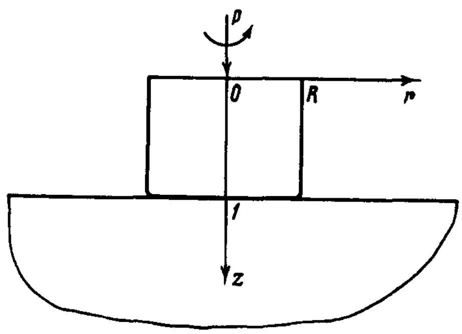

Fig. 1.

the initial conditions

$$\tau=0: \quad t=T=0, \quad f=\frac{\partial f}{\partial \tau}=0 \tag{1.6}$$

Here $k=(\lambda+\mu)/\mu=1/(1-2\nu)$, ν is Poisson's ratio, $\beta=(3\lambda+2\mu)\alpha_T$, λ and μ are Lamé coefficients, α_T is the temperature coefficient of linear expansion, $f(\tau)$ is a function of the punch settling, λ_1 and λ_2 are the thermal conductivities of the cylindrical punch and the half-space, respectively, $\gamma_0=\gamma_\alpha=\gamma_H$ are the heat-transfer coefficients with the external medium for the end of the cylinder, the side surface of the cylinder and the free surface of the half-space, respectively, T is the temperature of the cylindrical punch, t is the temperature of the half-space, $X_i=1/a_i$ $(i=1,2)$, where a_i are the thermal diffusivities of the cylinder and the half-space, respectively and h^{-1} is the thermal resistance coefficient. It was assumed in [4] that $\tau_{z\vartheta}=f_T\sigma_{zz}$ where f_T is the friction coefficient.

The functions t, T, u_r, u_z, σ_{zz}, τ_{rz} depend on three real variables (r, z and τ). Assuming that the curvature of the edge of the punch has a parabolic form, it is convenient to represent the function ($f(r,\tau)$ in the form [5]

$$f(r,\tau)=f_1(\tau)-\frac{(r-r^*)^2}{2R_p}H(r-r^*) \tag{1.7}$$

where $r^*<R$ is a certain point close to R (we assume that the curvature begins fairly close to the edge of the punch) and R_p is the radius of curvature of the edge.

When solving the problem we will also use the condition of dynamic equilibrium of the punch

$$m\frac{\partial^2 f_1(\tau)}{\partial \tau^2}=P(\tau)+2\pi\int_0^R r\sigma_{zz}(r,l,\tau)dr \tag{1.8}$$

2. FORMULATION OF THE PROBLEM IN TERMS OF LAPLACE TRANSFORMANTS

Applying a Laplace transformation to Eqs (1.1) and (1.2) and the boundary conditions (1.3)–(1.5), and using (1.6), we obtain

$$\Delta u_r^L-\frac{u_r^L}{r^2}+k\frac{\partial e^L}{\partial r}=\frac{\beta}{\mu}\frac{\partial t^L}{\partial r}, \quad \Delta u_z^L+k\frac{\partial e^L}{\partial z}=\frac{\beta}{\mu}\frac{\partial t^L}{\partial z} \tag{2.1}$$

$$\Delta T^L=X_1 sT^L, \quad \Delta t^L=X_2 st^L \tag{2.2}$$

$$z=0: \ \frac{\partial T^L}{\partial z}=\gamma_0 T^L \ (0\leqslant r\leqslant R); \quad r=R: \ \frac{\partial T^L}{\partial r}=-\gamma_\alpha T^L \ (0\leqslant z\leqslant l) \tag{2.3}$$

$$z=l: \ \lambda_2\frac{\partial t^L}{\partial z}-\lambda_1\frac{\partial T^L}{\partial z}=f_T\omega r\sigma_{zz}^L, \quad \lambda_2\frac{\partial t^L}{\partial z}+\lambda_1\frac{\partial T^L}{\partial z}=h(t^L-T^L)\,(0\leqslant r\leqslant R) \tag{2.4}$$

$$\frac{\partial t^L}{\partial z} = \gamma_H t^L \ (r > R)$$

$$z = l:\ u_z^L = f^L \ (0 \le r \le R), \quad \sigma_{zz}^L = 0 \ (r \ge R), \quad \tau_{rz}^L = 0 \ (r < \infty) \tag{2.5}$$

The functions t^L, T^L, u_r^L, u_z^L, σ_{zz}^L, τ_{rz}^L depend on two real variables (r and z) and one complex variable s. We will write Eq. (1.8), taking conditions (1.6) for the settling function into account, in the form

$$ms^2 f_1^L(s) = P^L(s) + 2\pi \int_0^R r\sigma_{zz}^L(r, l, s)dr \tag{2.6}$$

3. SOLUTION OF THE HEAT CONDUCTION PROBLEM FOR A PUNCH IN TERMS OF LAPLACE TRANSFORMANTS

In the first equation of (2.2) we will change to dimensionless coordinates $\rho = r/R$, $\zeta = z/l$. Using the finite-difference approximation of this equation and the second boundary condition of (2.3) with respect to the dimensionless radial coordinate ρ, and also the method of straight lines [6] at each of the N points of subdivision $\rho_i = \Delta\rho(i-1) = (i-1)/(N-1)$ $(i = 1, \ldots, N)$, we can reduce the problem for a cylinder to a system of linear differential equations, which can be represented in the form

$$d\mathbf{w}/d\zeta = B\mathbf{w} \tag{3.1}$$

$$\mathbf{w}^T(\zeta, s) = \left(T^L(\rho_1, \zeta, s), \ldots, T^L(\rho_N, \zeta, s), \frac{dT^L(\rho_1, \zeta, s)}{d\zeta}, \ldots, \frac{dT^L(\rho_N, \zeta, s)}{d\zeta} \right)$$

$$B = \begin{Vmatrix} O & E \\ B_1 & O \end{Vmatrix}$$

$$B_1 = \begin{Vmatrix} b_2(4 + b_3 s), -4b_2, 0, \ldots & & & , 0 \\ \vdots & & & i\updownarrow \\ \underbrace{0, \ldots, 0}_{i-2}, b_2\left(\frac{1}{2(i-1)} - 1\right), & b_2(2 + b_3 s), & -b_2\left(\frac{1}{2(i-1)} + 1\right), & 0, \ldots, 0 \\ \vdots & & & \vdots \\ 0, \ldots & & & , 0, -2b_2, b_2(2 + b_3 s) + b_4 \end{Vmatrix}$$

$$b_2 = \frac{l^2}{R^2 \Delta\rho^2}, \quad b_3 = R^2 \Delta\rho^2 X_1, \quad b_4 = \frac{\gamma_\alpha}{R}\left(1 + \frac{2}{\Delta\rho}\right)$$

where O is the zero matrix and E is the identity matrix (each of the matrices O, E and B_1 has dimensions of $N \times N$).

The solution of (3.1) can be constructed using a matrix exponential function [7]

$$\mathbf{w}(\zeta, s) = \exp(B(s)\zeta)\mathbf{d}(s), \quad \exp(B(s)\zeta) = \sum_{n=0}^{\infty} \frac{B^n(s)\zeta^n}{n!} \tag{3.2}$$

The function $\mathbf{d}(s)$ is determined using the boundary conditions.

4. SOLUTION OF THE PROBLEM FOR A HALF-SPACE IN DOUBLE LAPLACE AND HANKEL TRANSFORMANTS

Applying a Hankel transformation of zero order with respect to the coordinate r to the second equation of (2.2) and taking into account the fact that $t \to 0$ as $z \to \infty$ we can write the solution for the double transformant of the temperature in the form

$$t^{LH}(\xi, z, s) = D(\xi, s)\exp(-\psi_2(\xi, s)(z - l)), \quad \psi_2(\xi, s) = \sqrt{\xi^2 + X_2 s} \tag{4.1}$$

Using the solution of the axisymmetric equations of thermoelasticity for the half-space in Hankel transforms [3] and Eq. (4.1), we obtain expression for the double transformants of the stresses, displacements and heat flux. Using the last condition of (2.5) these expressions can be written in terms of the two unknown functions $C_1(\xi, s)$ and $D(\xi, s)$. In particular, on the boundary of the half-space we have

$$\begin{gathered}
t^{LH}(\xi,l,s)=D(\xi,s), \quad q^{LH}(\xi,l,s)=\lambda_2\psi_2(\xi,s)D(\xi,s)\\
u_z^{LH}(\xi,l,s)=\xi^2\vartheta_1 C_1(\xi,s)-\vartheta_3\psi_2(\xi,s)s^{-1}D(\xi,s)\\
\sigma_{zz}^{LH}(\xi,l,s)=\xi^3\sigma_1 C_1(\xi,s)-\sigma_3\xi s^{-1}(\psi_2(\xi,s)(k-1)^{-1}+\xi)D(\xi,s)\\
\vartheta_1=k\frac{k+1}{k-1}, \quad \vartheta_3=\frac{b_1}{X_2}\frac{k+1}{k-1}, \quad \sigma_1=-2\mu\frac{k^2}{k-1}, \quad \sigma_3=-\frac{2\mu b_1}{X_2}
\end{gathered} \tag{4.2}$$

5. DERIVATION OF THE SOLUTIONS FORA CYLINDER AND A HALF-SPACE IN ACCORDANCE WITH THE BOUNDARY CONDITIONS

Changing to the dimensionless parameter $\eta = \xi R$ in (4.2), applying an inverse Hankel transformation and satisfying the boundary conditions (2.4) and (2.5), we obtain the equations

$$\begin{gathered}
\frac{\lambda_2}{R^3}\int_0^\infty \Psi_2(\eta,s)F(\eta,s)\eta J_0(\eta\rho)d\eta+\frac{\lambda_1}{l}\frac{\partial T^L(\rho,\zeta,s)}{\partial\zeta}\bigg|_{\zeta=1}=-f_T\omega\rho R\sigma_{zz}^L(\rho,1,s)\\
\left(\frac{\lambda_1}{l}\frac{\partial T^L(\rho,\zeta,s)}{\partial\zeta}+hT^L(\rho,\zeta,s)\right)\bigg|_{\zeta=1}=\frac{1}{R^3}\int_0^\infty \lambda_2(\Psi_2(\eta,s)+hR)F(\eta,s)\eta J_0(\eta\rho)d\eta
\end{gathered} \tag{5.1}$$

$$\begin{gathered}
\int_0^\infty [\eta^2\vartheta_1 C(\eta,s)-\vartheta_3 Rs^{-1}\Psi_2(\eta,s)F(\eta,s)]\eta J_0(\eta\rho)d\eta=R^4 f^L(s) \quad (0\leqslant\rho\leqslant 1)\\
-\frac{1}{R^3}\int_0^\infty (\Psi_2(\eta,s)+\gamma_H R)F(\eta,s)\eta J_0(\eta\rho)d\eta=0 \quad (\rho\geqslant 1)\\
\frac{1}{R^5}\int_0^\infty [\eta^2\sigma_1 C(\eta,s)-\sigma_3 R\eta s^{-2}(\Psi_2(\eta,s)+\eta s)F(\eta,s)]\eta J_0(\eta\rho)d\eta=0
\end{gathered} \tag{5.2}$$

where $\Psi_2(\eta, s) = \sqrt{(\eta^2 + X_2 sR^2)}$, $C(\eta, s) = C_1(\eta/R, s)$, $F(\eta, s) = D(\eta/R, s)$.

We extend (5.2) over the whole ρ axis using the Heaviside function

$$\begin{gathered}
-\frac{1}{R^3}\int_0^\infty (\Psi_2(\eta,s)+\gamma_H R)F(\eta,s)\eta J_0(\eta\rho)d\eta=\varphi(\rho,s)H(1-\rho)\\
\frac{1}{R^5}\int_0^\infty [\eta^2\sigma_1 C(\eta,s)-\sigma_3 R\eta s^{-2}(\Psi_2(\eta,s)+\eta s)F(\eta,s)]\eta J_0(\eta\rho)d\eta=\sigma_{zz}^L(\rho,1,s)H(1-\rho)
\end{gathered} \tag{5.3}$$

After this we represent the unknown functions $\varphi(\rho, s)$ and $\sigma_{zz}^L(\rho, 1, s)$ in the form of expansions in Fourier–Bessel series with coefficients which depend on s

$$\varphi(\rho,s)=b_0(s)+\sum_{n=1}^{N-1} b_n(s)J_0(\mu_n\rho), \quad \sigma_{zz}^L(\rho,1,s)=\sum_{n=1}^{N} a_n(s)J_0(\mu_n\rho) \tag{5.4}$$

where μ_n are the zeros of the Bessel function of zero order $J_0(x)$.

Applying a Hankel transformation to (5.3), the unknown functions $F(\eta, s)$ and $C(\eta, s)$ can be expressed in terms of the function $a_n(s)$, $b_n(s)$ ($n = 1, \ldots, N$). Then, by satisfying relation (2.6) and also (5.1) and the first condition of (2.3) (relations (5.2) with representation (5.3) are satisfied automatically) at each point of the uniform subdivision $\rho_i = \Delta\rho(i-1) = (i-1)/(N-1)$ ($i = 1, \ldots, N$), we arrive at the system of equations

$$
\begin{aligned}
&\sum_{n=1}^{N} a_n(s)\vartheta_1 R\sigma_1^{-1}\mu_n J_1(\mu_n)\Lambda_n(\rho_i)+b_0(s)s^{-1}\Pi^*(\rho_i,s)+\\
&+\sum_{n=1}^{N-1} b_n(s)s^{-1}\mu_n J_1(\mu_n)\Pi_n(\rho_i,s)-f_2^L(\rho_i,s)=0\\
&\sum_{n=1}^{N} a_n(s)f_T\rho_i\omega R J_0(\mu_n\rho_i)+b_0(s)\lambda_2 I^*(\rho_i,s)+\\
&+\sum_{n=1}^{N-1} b_n(s)\lambda_2\mu_n J_1(\mu_n)I_n(\rho_i,s)+\lambda_1 l^{-1}\sum_{n=1}^{2N} M^*_{N+i,n}(s)d_n(s)=0\\
&b_0(s)Y^*(\rho_i,s)+\sum_{n=1}^{N-1} b_n(s)\mu_n J_1(\mu_n)Y_n(\rho_i,s)-\\
&-\lambda_1 l^{-1}\sum_{n=1}^{2N} M^*_{N+i,n}(s)d_n(s)-h\sum_{n=1}^{2N} M^*_{i,n}(s)d_n(s)=0\\
&l^{-1}d_{N+i}(s)-\gamma_0 d_i(s)=0\\
&2\pi R^2\sum_{n=1}^{2N} a_n(s)J_1(\mu_n)\mu_n^{-1}-ms^2 f_1^L(s)=-P^L(s)
\end{aligned}
\tag{5.5}
$$

Here ($i = 1, \ldots, N$)

$$\Lambda_n(\rho_i)=\int_0^\infty J_0(\eta)J_0(\eta\rho_i)(\mu_n^2-\eta^2)^{-1}d\eta$$

$$\Pi^*(\rho_i,s)=\int_0^\infty \Pi(\rho_i,s)J_1(\eta)d\eta,\quad \Pi_n(\rho_i,s)=\int_0^\infty \Pi(\rho_i,s)\eta(\mu_n^2-\eta^2)^{-1}J_0(\eta)d\eta$$

$$\Pi(\rho_i,s)=(\vartheta_1\sigma_3\sigma_1^{-1}(\Psi_2(\eta,s)(k-1)^{-1}+\eta)-\vartheta_3\Psi_2(\eta,s)(\Psi_2(\eta,s)+R\gamma_H)^{-1}J_0(\eta\rho_i)$$

$$I^*(\rho_i,s)=\int_0^\infty I(\rho_i,s)J_1(\eta)d\eta,\quad I_n(\rho_i,s)=\int_0^\infty I(\rho_i,s)\eta(\mu_n^2-\eta^2)^{-1}J_0(\eta)d\eta$$

$$I(\rho_i,s)=\Psi_2(\eta,s)(\Psi_2(\eta,s)+R\gamma_H)^{-1}J_0(\eta\rho_i)$$

$$Y^*(\rho_i,s)=\int_0^\infty Y(\rho_i,s)J_1(\eta)d\eta,\quad Y_n(\rho_i,s)=\int_0^\infty Y(\rho_i,s)\eta(\mu_n^2-\eta^2)^{-1}J_0(\eta)d\eta$$

$$Y(\rho_i,s)=(\lambda_2\Psi_2(\eta,s)+hR)(\Psi_2(\eta,s)+R\gamma_H)^{-1}J_0(\eta\rho_i)$$

and $M^*_{i,n}(s)$ are the elements of the matrix (3.2) when $\zeta = 1$.

Note that in this paper we have used the fact that $r^* = (1 - 1/(N-1))R$. Hence

$$f_2^L(\rho_i,s)=f_1^L(s)\ (i=1,\ldots,N-1),\quad f_2^L(\rho_N,s)=f_1^L(s)+\frac{(R\rho_N-r^*)^2}{2R_p s}$$

In addition to $f_1^L(s)$, in system (5.5) there are $4N$ unknown functions, namely

$$a_n(s),\ b_n(s)\ (n=1,\ldots,N) \tag{5.6}$$

the coefficients of the expansions of the unknown functions in Fourier–Bessel series and

$$d_n(s)\ (n=1,\ldots,2N) \tag{5.7}$$

the components of the vector $\mathbf{d}(s)$.

6. DETERMINATION OF THE ORIGINALS OF THE REQUIRED QUANTITIES

The transformant of only one of the required quantities (we have in mind the transformant $f_1^L(s)$ of the displacements of the elastic half-space under the plane part of the punch) is included directly in system (5.5). The transformants of the other required quantities are expressed in terms of the functions (5.6) and (5.7). Thus, on the boundary of the half-space

$$t^L(r,l,s)=b_0(s)R\int_0^\infty (\Psi_2(\eta,s)+R\gamma_H)^{-1}J_1(\eta)J_0\left(\eta\frac{r}{R}\right)d\eta+$$
$$+\sum_{n=1}^{N-1} b_n(s)\mu_n J_1(\mu_n)R\int_0^\infty (\Psi_2(\eta,s)+R\gamma_H)^{-1}(\mu_n^2-\eta^2)^{-1}\eta J_0(\eta)J_0\left(\eta\frac{r}{R}\right)d\eta \tag{6.1}$$

$$q^L(r,l,s)=b_0(s)\lambda_2 I^*\left(\frac{r}{R},s\right)+\sum_{n=1}^{N-1} b_n(s)\mu_n J_1(\mu_n)\lambda_2 I_n\left(\frac{r}{R},s\right) \tag{6.2}$$

$$u_z^L(r,l,s)=\sum_{n=1}^{N} a_n(s)\vartheta_1 R\sigma_1^{-1}\mu_n J_1(\mu_n)\Lambda_n\left(\frac{r}{R}\right)+b_0(s)s^{-1}\Pi^*\left(\frac{r}{R},s\right)+$$
$$+\sum_{n=1}^{N-1} b_k(s)s^{-1}\mu_n J_1(\mu_n)\Pi_n\left(\frac{r}{R},s\right) \tag{6.3}$$

The transformants of the temperature and the heat flux in the cylinder can be found from (3.2), while the transformant of the contact stresses can be found from (5.4).

To calculate the originals of these functions we will use a numerical method of inverting the Laplace transformation employing Fourier sine series [8]. We will illustrate this using the example of the function $f_1(\tau)$. Using this method we obtain

$$f_1(\tau)=\sum_{k=0}^{\infty}\sin[(2k+1)\vartheta]e_k,\quad \vartheta=\arccos(e^{-\sigma\tau}) \tag{6.4}$$

where σ is a certain real positive number which is chosen depending on the range for which one must obtain the value of the original, and e_k are constant coefficients, to calculate which, if we are limited in (6.4) to the first N^* terms of the series, we have the following system of linear algebraic equations with triangular matrix

$$\sum_{k=0}^{n}\frac{2k+1}{2n+1}C_{2n+1}^{n-k}e_k=\frac{4^{n+1}}{\pi}\sigma f_1^L[(2n+1)\sigma]\quad (n=0,1,\ldots,N^*) \tag{6.5}$$

We obtain the quantities $f_1^L(s)$ required in order to use the method at differently situated points on the real axis by solving system (5.5) for each of the $s_n = (2n+1)\sigma$ $(n - 0, 1, \ldots, N^*)$.

7. ANALYSIS OF THE NUMERICAL RESULTS

It is assumed that the force P acting on the punch is given by the equation

$$P(\tau)=F(1-\exp(-a\tau)) \tag{7.1}$$

where a and F are certain constants.

When carrying out the calculations we also assumed that the material of the cylinder is steel while the material of the half-space is aluminium. The values of the constants were chosen as follows: $N = 17$, $N^* = 11$, $\gamma_H = \gamma_0 = \gamma_\alpha = 10\ \mathrm{m}^{-1}$, $h = 10\ \mathrm{kW/m^2\,K}$, $\alpha_T = 22.9\times10^{-6}\ \mathrm{K}^{-1}$, $F = 30$ kN, $\omega = 0.5\ \mathrm{s}^{-1}$, $R = 1$ m, $l = 0.2$ m, $X_1 = 200{,}000\ \mathrm{s/m^2}$, $X_2 = 50{,}000\ \mathrm{s/m^2}$, $a = 1$, $\lambda = 5.46\times10^{10}$ Pa, $\mu = 2.56\times10^{10}$ Pa, $f_T = 0.1$, $\lambda_1 = 22$ W/m.K, $\lambda_2 = 209$ W/m K and $m = 4900$ kg.

The results obtained are partially shown in Figs 2–6. In Figs 2 and 3 we show the distribution of the contact stresses and temperature, respectively, on the boundary of the half-space at different instants of time. In Fig. 4 we show the changes with time of the displacements of points of the elastic half-space which are in contact with the punch (the function $f_1(\tau)$ is shown on a scale of $1{:}0.179\times10^{-3}$ m), and the temperature (on a scale of 1:28.5°) at points of the boundary of the half-space close to the edge of

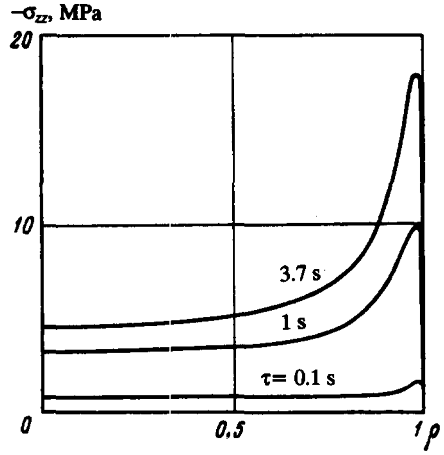

Fig. 2.

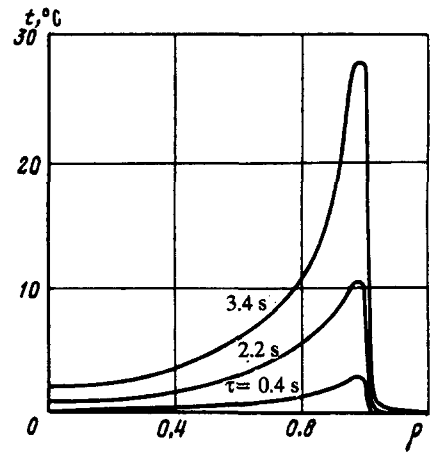

Fig. 3.

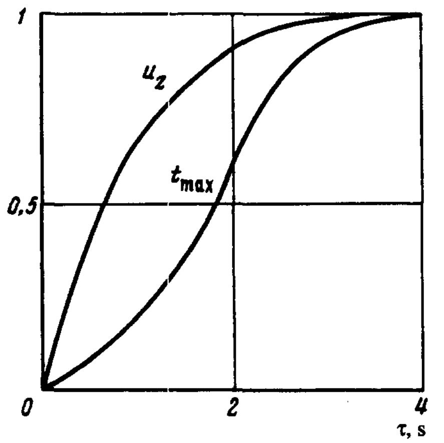

Fig. 4.

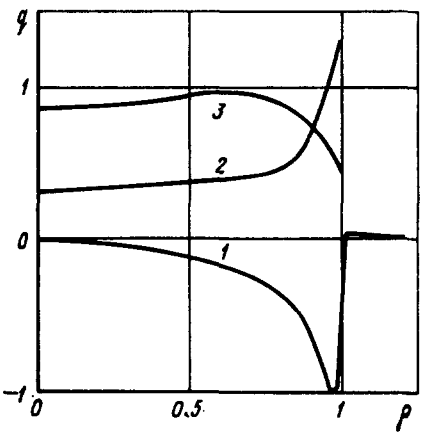

Fig. 5.

the punch. If we follow the changes in the other required quantities, we find that, after a certain instant of time, they hardly change. Taking as the time required to reach the steady state τ_c the time after which any characteristic investigated does not change by more than 1%, we obtain $\tau_c \approx 3.7$ s. In the steady state the heat fluxes shown in Fig. 5 are established. Curve 1 corresponds to points of the half-space close to the lower face of the punch, and curves 2 and 3 are drawn for points of the upper and lower faces of the punch, respectively. We chose the following scales for curves 1–3, respectively—1:10^6 W/m^2, 1:5 × 10^3 W/m^2, and 1:2 × 10^3 W/m^2. The contact area remains unchanged all the time.

In order to investigate the effect of the function $P(\tau)$ on the characteristics of the steady process we carried out calculations for various values of the parameter a. We established the following results.

1. The distribution of the stresses, displacements, temperature and heat fluxes under steady conditions do not, in fact, depend on a. The disagreement for values of a = 0.5, 1, and 3 amounted to no more than 6%.

2. When a is reduced from 1 to 0.5 the time at which a steady state is reached increases from 3.7 s to 6 s. However, when there is a considerable increase in the value of a, this means sharper action of the force $P(\tau)$, and no reduction in τ_c is observed. In Fig. 6 we show graphs of the vertical displacements under the punch for various values of a. The dashed curve is drawn for a stress $P(\tau) = FH(\tau)$.

3. When the punch is loaded with a force given by (7.1), the following temperatures are established at each point of the bodies considered

$$t(\rho, z, \tau) \leqslant t_c(\rho, z), \qquad T(\rho, z, \tau) \leqslant T_c(\rho, z)$$

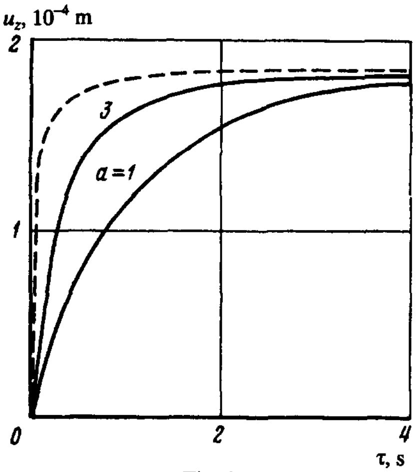

Fig. 6.

where $t_c(\rho, z)$ and $T_c(\rho, z)$ are the solutions of the steady contact problem.

To estimate the accuracy of the method employed, the characteristics of the steady state were compared with the solutions of the corresponding steady problem [2]. The maximum disagreements were as follows: for the contact stresses 4%, for the temperature and heat flux on the boundary of the half-space, 3% and 5%, respectively, and for the temperature at the bottom of the punch 3%.

We also analysed the effect of the curvatures of the edge of the punch on the results obtained. To do this, the subdivision of the interval (0, 1) was doubled, while the part with curvature was reduced by half. The contact stresses and the temperature on the plane part of the punch changed only slightly.

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